2. PATTERNS, FUNCTIONS & ALGEBRA: PROGRESSION – FP to IP

TOPICS	GRADE 3		GRADE 4
2.1. Number Patterns	2.2	**Copy, extend & describe** • Simple number sequences to at least 999 **Create & describe own patterns**	**Investigate & extend patterns** • *Investigate & extend numeric patterns looking for relationships or rules of patterns:* - Sequences involving a constant difference or ratio - Of learner's own creation • *Describe observed relationships or rules in learner's own words* **Input & Output values** • *Determine input values, output values and rules for patterns & relationships using flow diagrams* **Equivalent forms** • *Determine equivalence of different descriptions of the same relationship or rule presented* - Verbally - In a flow diagram - By a number sentence
2.2. Geometric Patterns	2.1	**Copy, extend and describe** In words: • Simple patterns made with physical objects • More complex patterns made with drawings of lines, shapes or objects **Create & describe own patterns** • With physical objects • By drawing lines, shapes or objects **Patterns all around us** Identify, describe in words and copy geometric patterns • In nature • From modern everyday life • From our cultural heritage	**Investigate & extend** • *Geometric patterns looking for relationships or rules of patterns* - Represented in physical or diagrammatic form - Sequences not limited to a constant difference or ratio - Of learner's own creation • *Describe observed relationships or rules in learner's own words* **Input & Output values** • *Determine input values, output values and rules for patterns & relationships using flow diagrams* **Equivalent forms** • Determine equivalence of different descriptions of the same relationship or rule presented - Verbally - In a flow diagram - By a number sentence
2.3. Number Sentences Intro to Algebraic Expressions			**Number sentences** • *Write number sentences to describe problem situations* • *Solve & complete number sentences by:* - Inspection - Trial & improvement - substitution

Patterns have elements that alternate, repeat, increase or decrease in a regular way. Describing the regular relationships should lead to predicting how the patterns will continue. Recognising that patterns are predictable is an important mathematical concept. The ability to generalise about patterns and to use the obtained knowledge to predict unknown knowledge is a powerful aspect of patterning. In order to generalise and predict, learners have to move from observing patterns as sequences of "what comes next" to structure analysis of the patterns i.e. viewing patterns as combinations of repeating units.

Young learners need to be involved in activities involving thinking about the predictability and structure of patterns so that their ideas about patterning connect effectively with algebraic thought later in the higher grades.

Children develop language and speech before number concepts and should thus be provided with a mathematical vocabulary to begin with. Numerical symbols & signs will only be understood if the child has a thorough grasp of the language to describe observations and relationships. Vocabulary is acquired as a result of observation, investigation and experimentation with apparatus, pictures and real life experiences. Discussions ensure that the child derives full benefits from effective learning experiences. For effective knowledge and skill development in patterns and algebra, learners have to acquire vocabulary in the language of size, space, position, shape, transformation and pattern. Learners should be allowed to use their own informal language in descriptions and discussions. The teacher should, however encourage the development and use of relevant formal terminology.

Numerical patterns and expressions are dealt with in familiar and meaningful number contexts by reflecting and verifying procedures through calculations. Geometrical patterns in nature, culture and the environment are copied, created, investigated and extended to develop algebraic thinking, reasoning, skills and knowledge.

(Economopoulos, K. (1998). Teaching Children Arithmetic).

Complete and Extend Patterns

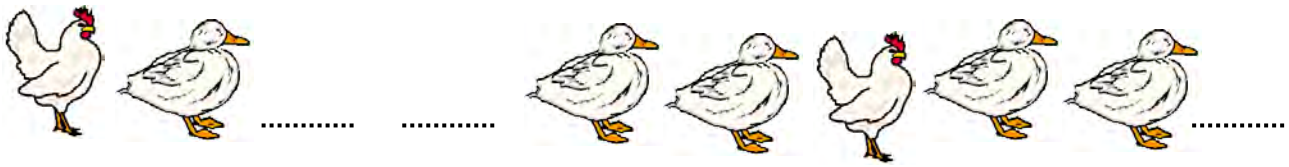
Work with a partner.

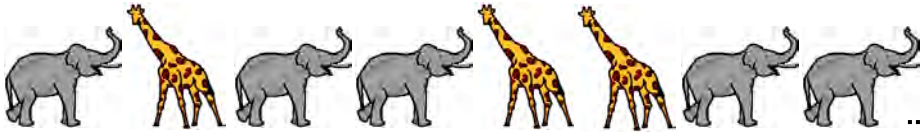
Tell your class what are missing in each pattern.

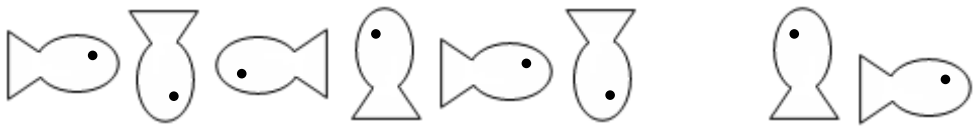
Explain your answers.

1.  ....

2.  ....

3.  ....

4.  .... 

5.  ....

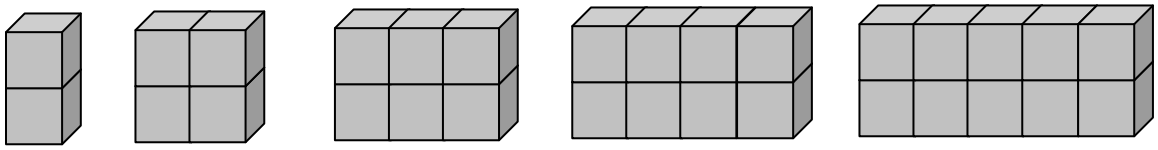
Terminology: pattern, between, before, after, next, rotate, clockwise, turn, reflect, etc.

Cubes and Number Patterns

Work with a partner. Use cubes to build each pattern.

Write down the number of cubes in each pattern sequence. Write the next three numbers in each pattern.

(a)



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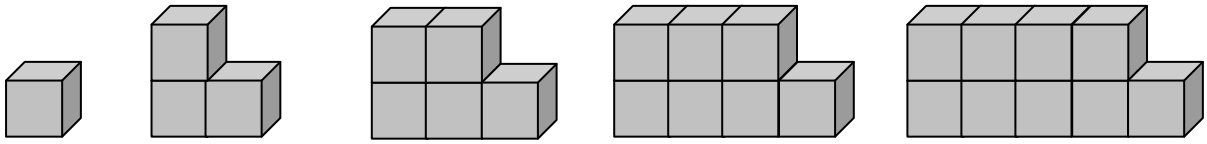
(b)



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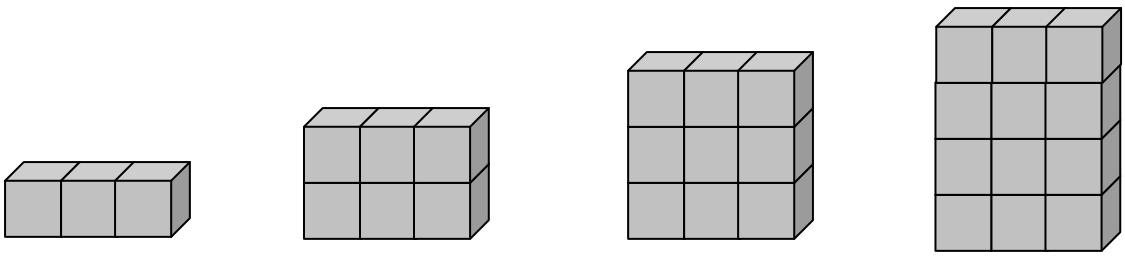
(c)



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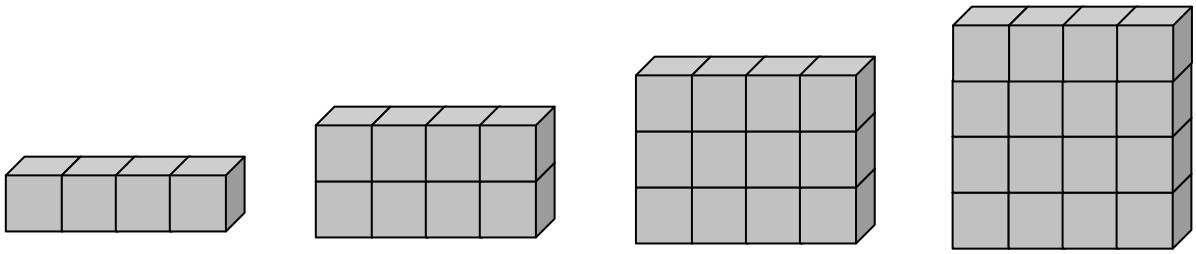
(d)



……. ……. ……. ……. ……. ……. …….

Share your solutions with the class.

Terminology: multiples, cubes, sequence, even, uneven, odd.

Patterns in a 100-grid

Use individual 100-grid copies and a large grid in the class for learners to identify, copy and describe number patterns according to their levels of development:

- Even and uneven (odd) numbers.
- Multiples of numbers
- Count on e.g. 10 from any number.
- Horizontal patterns (in rows)
- Vertical patterns (in columns)
- Diagonal patterns
- Patterns within squares

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	76	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	76	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Cut 100-grids on the thick lines. Ask learners to fit the pieces together to make the original grid. They have to identify patterns.

Also identify:
- Square numbers
- Triangular numbers
- Prime numbers
- Factors, etc.

Terminology: even, uneven, odd, horizontal, vertical, multiples, diagonal, square numbers, triangular numbers, prime numbers, factors, etc.

Patterns in Multiples

- Identify the patterns in the multiples of numbers.
- Fill in the missing numbers.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	---	32	36
5	10	15	20	25	30	35	40	---
---	12	18	---	30	36	42	48	54
7	14	21	28	35	42	49	56	63
---	---	---	---	---	---	---	---	---
9	18	27	36	45	54	63	72	81
10	20	30	40	50	60	70	80	90
11	22	33	---	55	---	77	---	99
12	---	---	48	60	72	---	96	108
13	26	39	52	65	78	91	104	117
14	28	42	56	70	84	98	112	126
15	30	45	60	75	90	105	120	135
---	---	---	---	---	---	---	---	---

Terminology: even, uneven, odd, horizontal, vertical, multiples, factors, etc.

Patterns in Number Operations

A. Complete the calculations.

8 × 8 = …… **6 × 6 = ……**

9 × 7 = …… **7 × 5 = ……**

Will the pattern work every time?

Write down more calculations to find out.

B. Calculate the following.

9 × 5 = ……. **9 × 6 = …….**

9 × 4 = ……. **9 × 7 = …….**

9 × 3 = ……. **9 × 8 = …….**

9 × 2 = ……. **9 × 9 = …….**

9 × 1 = ……. **9 × 10 = …….**

Tell the class what you notice.

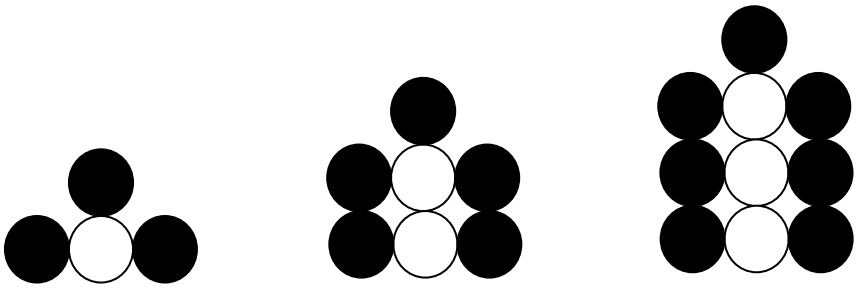
This way of displaying the multiplication table and the emphasis placed on its characteristic pattern will facilitate its memorisation.

Growing Patterns 1

Nalini, Gary and Sipho work as a team to earn pocket money. They make necklaces with pendants, which they sell at the festival.

They use different colour beads and arrange them in various patterns.

Here is one of the bead arrangements they use.



Work with your group.

Use two different colour counters to create the patterns.

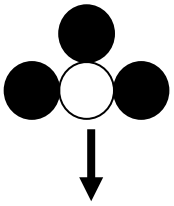
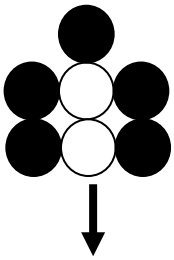
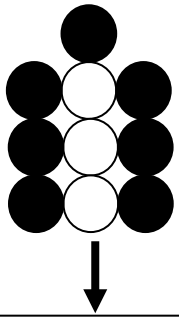
Describe the patterns.

Now create the next three patterns in the sequence.

Make drawings of the patterns.

Growing Patterns 2

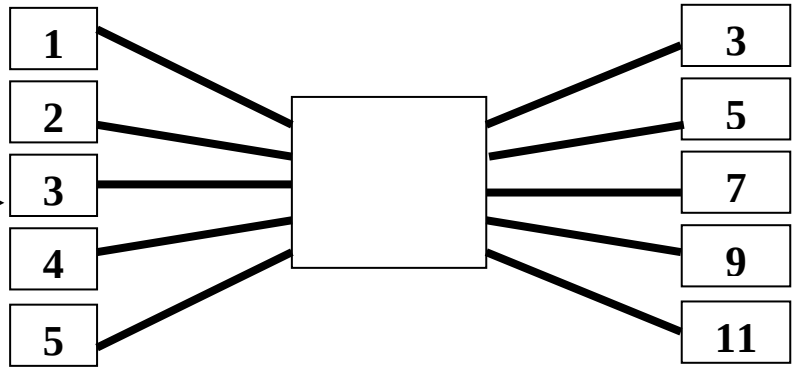
We can describe the patterns and represent the numbers like this.

	Pattern 1	Pattern 2	Pattern 3
			
In words	There are 3 blacks beads for a white bead	There are 5 black beads for 2 white beads	There are 7 black beads for 3 white beads
	1 white bead 3 black beads	2 white beads 5 black beads	3 white beads 7 black beads

In a table

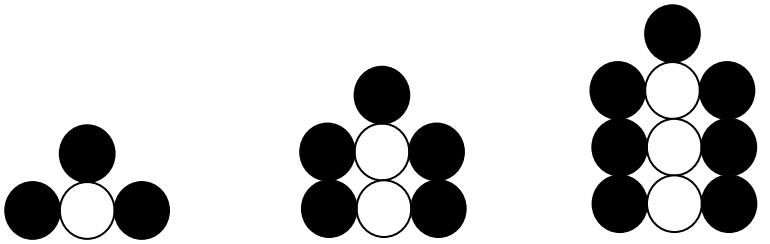
White beads	1	2	3	4	5	6	7	8
Black beads	3	5	7	9	11	13	15	17

In a flow diagram



Growing Patterns 3

We can also describe the patterns and represent the numbers like this.



	Pattern 1	Pattern 2	Pattern 3
In number expressions → White beads:	1	1 + 1	1 + 2
Blacks beads:	1 + 1 + 1	2 + 2 + 1	3 + 3 + 1
	2 × 1 + 1	2 × 2 + 1	3 × 2 + 1

Pattern 4 → White beads: 4 Black beads: $4 \times 2 + 1 = 9$

In algebraic expressions →

If you are asked to work out the number of beads in the 10^{th} bead pattern, you will need a rule. You cannot go on building patterns or use drawings to get the answer. We now know that, to work out the number of black beads for any number of white beads, you have to multiply the number of white beads by 2 and add 1.

	White beads	Black beads
Pattern 5 →	5	$5 \times 2 + 1 = 11$
Pattern 6 →	6	$6 \times 2 + 1 = 13$
Pattern 7 →	7	$7 \times 2 + 1 = 15$
Pattern 10 →	10	$10 \times 2 + 1 = 21$

Any number of white beads (w) $\quad w \times 2 + 1 = 2w + 1$